



# Fermi National Accelerator Laboratory

FERMILAB-PUB-95/107-T

OSU Preprint 303

May 1995

## An Explicit $SO(10) \times U(1)_F$ Model of the Yukawa Interactions

Carl H. ALBRIGHT

Department of Physics, Northern Illinois University, DeKalb, Illinois 60115\*

and

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510†

Satyanarayan NANDI

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078‡

### Abstract

We construct an explicit  $SO(10) \times U(1)_F$  model of the Yukawa interactions by using as a guide previous phenomenological results obtained from a bottom-up approach to quark and lepton mass matrices. The global  $U(1)_F$  family symmetry group sets the textures for the Majorana and generic Dirac mass matrices by restricting the type and number of Higgs diagrams which can contribute to each matrix element, while the  $SO(10)$  group relates each particular element of the up, down, neutrino and charged lepton Dirac matrices. The Yukawa couplings and vacuum expectation values associated with pairs of 1, 45, 10, and 126 Higgs representations successfully correlate all the quark and lepton masses and mixings in the scenario incorporating the nonadiabatic solar neutrino and atmospheric neutrino depletion effects.

PACS numbers: 12.15.Ff, 12.60Jv

---

\*Permanent address

†Electronic address: ALBRIGHT@FNALV

‡Electronic address: PHYSSNA@OSUCC



In a series of manuscripts [1], the authors have demonstrated how a new bottom-up approach to the quark and lepton mass and mixing problem can be used to construct phenomenological quark and lepton mass matrices at the supersymmetric  $SO(10)$  grand unification scale, which lead to the assumed experimental input at the low scales. As such, this procedure provides an alternative to the usual top-down approach [2], where mass matrices are constructed based on some well-defined theoretical concepts. Of special interest is a set of mass matrices found by our approach which exhibit a particularly simple  $SO(10)$  structure for the scenario based on the depletions of solar [3] and atmospheric [4] neutrinos through oscillations.

In this letter we construct an explicit  $SO(10) \times U(1)_F$  model at the grand unification scale by making use of the phenomenological mass matrices as a guide. The global  $U(1)_F$  family symmetry singles out a rather simple set of tree diagrams which set the textures for the Dirac and Majorana mass matrices, while  $SO(10)$  relates the corresponding up, down, neutrino and charged lepton Dirac matrix elements to each other. The quantitative numerical results obtained from the model agree in detail with the input data assumed for the bottom-up approach.

The starting point for our bottom-up approach was the reasonably well-known quark mass and Cabibbo-Kobayashi-Maskawa mixing matrix data [5]. To this we appended neutrino mass and mixing data consistent with the nonadiabatic Mikheyev-Smirnov-Wolfenstein (MSW) [6] resonant matter oscillation depletion [3] of the solar electron-neutrino flux together with atmospheric muon-neutrino depletion [4] through oscillations into tau-neutrinos. After running the Yukawa couplings to the grand unification scale, we applied Sylvester's theorem, as illustrated by Kusenko [7] for quark data alone, to reconstruct complex-symmetric mass matrices. The construction is clearly not unique, but one can vary two parameters which determine the weak bases in order to select a set of mass matrices which exhibit particularly simple  $SO(10)$  structure for as many matrix elements as possible. We refer the interested reader to Ref. [1] for details and begin here with the special phenomenological

matrices singled out by this procedure:

$$M^U \sim M^{N_{Dirac}} \sim \text{diag}(\overline{126}; \overline{126}; 10) \quad (1a)$$

$$M^D \sim M^E \sim \begin{pmatrix} 10', \overline{126} & 10', \overline{126}' & 10' \\ 10', \overline{126}' & \overline{126} & 10' \\ 10' & 10' & 10 \end{pmatrix} \quad (1b)$$

with  $M_{11}^D$ ,  $M_{12}^E$  and  $M_{21}^E$  anomalously small and only the 13 and 31 elements complex. Entries in the matrices stand for the Higgs representations contributing to those elements. Recall that the  $SO(10)$  product rules read

$$16 \times 16 = 10_s + 120_a + 126_s, \quad (2a)$$

$$16 \times \overline{16} = 1 + 45 + 210 \quad (2b)$$

We have assumed that vacuum expectation values (VEVs) develop only for the symmetric representations 10 and 126 and for 1 and 45. The Majorana neutrino mass matrix  $M^R$ , determined from the seesaw formula [8] with use of  $M^{N_{Dirac}}$  and the reconstructed light neutrino mass matrix, exhibits a nearly geometrical structure given by [9]

$$M^R \sim \begin{pmatrix} F & -\sqrt{FE} & \sqrt{FC} \\ -\sqrt{FE} & E & -\sqrt{EC} \\ \sqrt{FC} & -\sqrt{EC} & C \end{pmatrix} \quad (3)$$

where  $E = \frac{5}{6}\sqrt{FC}$  with all elements relatively real. It can not be purely geometrical, however, since the singular rank-1 matrix can not be inverted as required by the seesaw formula,  $M^{N_{eff}} \simeq -M^{N_{Dirac}}(M^R)^{-1}M^{N_{Dirac}^T}$ .

The challenge is now to introduce a family symmetry which will enable one to derive the mass matrix patterns in a simple fashion. For this purpose, we propose to use a global  $U(1)_F$  family symmetry [10] and to reduce the problem to the construction of one generic Dirac matrix,  $M_{Dirac}$ , along with the single Majorana matrix,  $M^R$ . As noted above, the  $SO(10)$  symmetry will relate the corresponding matrix elements of the four Dirac matrices to each other. The important roles played here by supersymmetry (SUSY) are twofold. Not

only does SUSY control the running of the Yukawa couplings between the SUSY GUT scale and the weak scale where it is assumed to be softly broken, but it also allows one to assume that only simple tree diagrammatic contributions to the mass matrices need be considered as a result of the nonrenormalization theorem applied to loop diagrams. This tree diagram procedure was first suggested by Dimopoulos [11] twelve years ago.

Simplicity of the  $SO(10)$  structure requires that just one Higgs 10 representation contributes to the  $(M_{Dirac})_{33}$  element (hereafter labeled D33), i.e., we assume complete unification of the Yukawa couplings at the unification scale:  $\bar{m}_\tau = \bar{m}_b = \bar{m}_t / \tan \beta_{10}$ , where  $\tan \beta_{10}$  is equal to the ratio of the up quark to the down quark VEVs in the 10

$$\begin{aligned}\bar{m}_t &= g_{10}(v/\sqrt{2}) \sin \beta_{10} \equiv g_{10}v_u \\ \bar{m}_b = \bar{m}_\tau &= g_{10}(v/\sqrt{2}) \cos \beta_{10} \equiv g_{10}v_d \\ \tan \beta_{10} &= v_u(5)/v_d(\bar{5})\end{aligned}\tag{4a}$$

in terms of the  $SU(5)$  decomposition of  $SO(10)$  with  $v = 246$  GeV. The same 10 can not contribute to  $D23 = D32$ , for the diagonal nature of  $M^U$  and  $M^{N_{Dirac}}$  requires the presence of another 10' with

$$\tan \beta_{10'} = v'_u(5')/v'_d(\bar{5}') = 0\tag{4b}$$

Likewise we assume a pure  $\overline{126}$  contribution to D22 with

$$\tan \beta_{\overline{126}} = w_u(5)/w_d(\overline{45})\tag{4c}$$

The tree diagrams for these  $M_{Dirac}$  matrix elements are illustrated in Fig. 1a.

We shall now assign  $U(1)_F$  charges to the three families (in order of appearance) and to the Higgs representations as follows with the numerical values to be determined later:

$$16_3^\alpha, 16_2^\beta, 16_1^\gamma, 10^a, 10'^b, \overline{126}^c\tag{5a}$$

Conservation of  $U(1)_F$  charges then requires  $2\alpha + a = 0$ ,  $\alpha + \beta + b = 0$  and  $2\beta + c = 0$  as seen from the diagrams in Fig. 1a.

In the above, we have taken the 2-3 sector of  $M_{Dirac}$  to be renormalizable with two  $10$ 's and one  $\overline{126}$  developing low scale VEVs. We assume the rest of the  $M_{Dirac}$  elements arise from non-renormalizable contributions with the leading ones shown in Fig. 1b. For D13 we introduce a  $45_X^e$  Higgs field and construct an explicitly complex-symmetric contribution with the dimension-6 diagram, for which  $U(1)_F$  charge conservation requires  $\alpha + \gamma + b + 2e = 0$ . This  $45_X$  Higgs field develops a VEV in the direction which breaks  $SO(10) \rightarrow SU(5) \times U(1)_X$  with the  $SU(5)$  subgroup remaining unbroken. For D12 we introduce a different  $45_Z^h$  Higgs field which breaks  $SO(10) \rightarrow \text{flipped } SU(5) \times U(1)$  and is related to the orthogonal  $45_X$  and  $45_Y$  hypercharge VEVs by

$$\langle 45_Z \rangle = \frac{6}{5} \langle 45_X \rangle - \frac{1}{5} \langle 45_Y \rangle \quad (6)$$

as given in Table I. This Higgs field contributes to  $M_{12}^D$  but not to  $M_{12}^E$ ; thus it generates a zero in this position for the charged lepton mass matrix as suggested in (1b). The  $U(1)_F$  charge conservation equation reads  $\beta + \gamma - b + 2h = 0$ , as the  $10'^*$  Higgs field is required here to reduce the number of contributing diagrams. The D11 element is dimension-8 or higher and is left unspecified. The complex-symmetric Yukawa diagrams which we wish to generate are then neatly summarized by the ordering of the Higgs fields:

$$\begin{aligned} D33 : & 16_3 - 10 - 16_3 \\ D23 : & 16_2 - 10' - 16_3 \\ D32 : & 16_3 - 10' - 16_2 \\ D22 : & 16_2 - \overline{126} - 16_2 \\ D13 : & 16_1 - 45_X - 10' - 45_X - 16_3 \\ D31 : & 16_3 - 45_X - 10' - 45_X - 16_1 \\ D12 : & 16_1 - 45_Z - 10'^* - 45_Z - 16_2 \\ D21 : & 16_2 - 45_Z - 10'^* - 45_Z - 16_1 \end{aligned} \quad (7a)$$

In order to obtain a different set of diagrams for the Majorana matrix, we begin the M33 contribution with a dimension-6 diagram shown in Fig. 1c by including a new  $\overline{126}'^d$

Higgs which develops a VEV at the GUT scale in the  $SU(5)$  singlet direction, along with a pair of  $1^g$  Higgs fields. Here  $2\alpha + d + 2g = 0$ . The nearly geometric structure for  $M^R$  can then be generated by appending more Higgs fields to each diagram. For M23 we introduce another  $1'^f$  Higgs field to construct a diagram with one  $\overline{126}'^d$ , one  $45_X^e$ , one  $1'^f$  and two  $1^g$  fields with charge conservation demanding  $\alpha + \beta + d + 2g + e + f = 0$ . The new  $1'$  field is needed in order to scale properly the Majorana matrix elements relative to each other. The remaining leading-order diagrams of the complex-symmetric Majorana mass matrix follow by appending more  $45_X^e$ ,  $45_Z^h$  and  $1'^f$  Higgs lines. The pattern is made clear from the charge conservation equations:  $2\beta + d + 2g + 2e + 2f = 0$  for M22,  $\alpha + \gamma + d + 2g + e + h + 2f = 0$  for M13,  $\beta + \gamma + d + 2g + 2e + h + 3f = 0$  for M12, and  $2\gamma + d + 2g + 2e + 2h + 4f = 0$  for M11.

In summary, the following Higgs representations have been introduced in addition to those in (5a):

$$\overline{126}'^d, 45_X^e, 45_Z^h, 1^g, 1'^f \quad (5b)$$

all of which generate massive VEVs near the GUT scale. In order to obtain CP-violation in the quark and lepton mixing matrices, we allow the VEVs for  $45_X$ ,  $45_Z$ ,  $1$  and  $1'$  to be complex, but the VEVs associated with the  $10$ ,  $10'$ ,  $\overline{126}$  and  $\overline{126}'$  representations can be taken to be real without loss of generality as seen from our bottom-up results. Clearly, many permutations of the Higgs fields are possible in the higher-order diagrams.

At this point a computer search was carried out to generate  $U(1)_F$  charge assignments leading to the fewest additional diagrams allowed by charge conservation. An especially interesting charge assignment stood out for which

$$\begin{aligned} \alpha &= 9, \beta = -1, \gamma = -8 \\ a &= -18, b = -8, c = 2, d = -22, e = 3.5, f = 6.5, g = 2.0, h = 0.5 \end{aligned} \quad (8a)$$

One should note that since  $\alpha + \beta + \gamma = 0$ , the  $[SO(10)]^2 \times U(1)_F$  anomaly vanishes, whereas the  $[U(1)_F]^3$  anomaly does not. Simplicity then suggests that the  $U(1)_F$  family symmetry group be global with a familon being generated upon its breaking.

With the above charge assignments we can greatly limit the number of permutations and eliminate other unwanted diagrams by restricting the  $U(1)_F$  charges appearing on the superheavy internal fermion lines. With the following minimum set of allowed charges for the left-handed superheavy fermions  $F_L$  and their mirror partners  $F_L^c$

$$\begin{aligned} F_L : & -0.5, \quad 1.0, \quad 2.0, \quad 4.0, \quad 4.5, \quad -4.5, \quad -7.5, \quad 11.0, \quad 12.5 \\ F_L^c : & 0.5, \quad -1.0, \quad -2.0, \quad -4.0, \quad -4.5, \quad 4.5, \quad 7.5, \quad -11.0, \quad -12.5 \end{aligned} \quad (8b)$$

we recover just the leading-order diagrams listed in (7a) for the generic Dirac mass matrix together with the following uniquely-ordered diagrams for the complex-symmetric Majorana mass matrix

$$\begin{aligned} M33 : & 16_3 - 1 - \overline{126}' - 1 - 16_3 \\ M23 : & 16_2 - 1 - 45_X - 1' - \overline{126}' - 1 - 16_3 \\ M32 : & 16_3 - 1 - \overline{126}' - 1' - 45_X - 1 - 16_2 \\ M22 : & 16_2 - 1 - 45_X - 1' - \overline{126}' - 1' - 45_X - 1 - 16_2 \\ M13 : & 16_1 - 45_X - 1' - 1 - 45_Z - 1' - \overline{126}' - 1 - 16_3 \\ M31 : & 16_3 - 1 - \overline{126}' - 1' - 45_Z - 1 - 1' - 45_X - 16_1 \\ M12 : & 16_1 - 45_X - 1' - 1 - 45_Z - 1' - \overline{126}' - 1' - 45_X - 1 - 16_2 \\ M21 : & 16_2 - 1 - 45_X - 1' - \overline{126}' - 1' - 45_Z - 1 - 1' - 45_X - 16_1 \\ M11 : & 16_1 - 45_X - 1' - 1 - 45_Z - 1' - \overline{126}' - 1' - 45_Z - 1 - 1' - 45_X - 16_1 \end{aligned} \quad (7b)$$

Several other higher-order diagrams are allowed by the  $U(1)_F$  charges given in (8a,b) and appear for D11, D22, M23 and M32 with the Higgs fields ordered as follows:

$$\begin{aligned} D11 : & 16_1 - 45_X - 1' - 1 - \overline{126} - 1 - 1' - 45_X - 16_1 \\ D22 : & 16_2 - 45_Z - 10'^* - 1'^* - 16_2, \quad 16_2 - 1'^* - 10'^* - 45_Z - 16_2 \\ M23 : & 16_2 - 45_X^* - 1' - 1 - 45_Z - 1' - \overline{126}' - 1 - 16_3 \\ M32 : & 16_3 - 1 - \overline{126}' - 1' - 45_Z - 1 - 1' - 45_X^* - 16_2 \end{aligned} \quad (7c)$$

These corrections to M23 and M32 ensure that  $M^R$  is rank 3 and nonsingular, so that the seesaw formula can be applied. Up to this point the contributions are all complex-symmetric.

Additional correction terms of higher order which need not be complex-symmetric can be generated for the Dirac and Majorana matrix elements, if one allows additional superheavy fermion pairs with new  $U(1)_F$  charges. Such a subset which does not destroy the pattern constructed above but helps to improve the numerical results given later consists of the following:

$$\begin{aligned} F_L : \quad & 1.5, \quad -6.0, \quad -6.5 \\ F_L^c : \quad & -1.5, \quad 6.0, \quad 6.5 \end{aligned} \tag{8c}$$

We shall enumerate the additional diagrams contributing to D11, D12, D13, D21, D31 and M11 in a more detailed paper in preparation [12].

We assume the superheavy fermions all get massive at the same mass scale, so each  $1, 1', 45_X$  or  $45_Z$  vertex factor can be rescaled by the same propagator mass  $M$ . As a result there are 14 independent parameters which can then be taken to be

$$\begin{aligned} & g_{10}v_u, \quad g_{10}v_d, \quad g_{10'}v'_d, \quad g_{126}w_u, \quad g_{126}w_d, \quad g_{126'}w' \\ & g_{45_X}u_{45_X}/M, \quad g_{45_Z}u_{45_Z}/M, \quad g_1u_1/M, \quad g_{1'}u_{1'}/M \\ & \phi_{45_X}, \quad \phi_{45_Z}, \quad \phi_1, \quad \phi_{1'} \end{aligned} \tag{9}$$

In addition, one needs the Clebsch-Gordan coefficient appearing at each vertex which can be read off from Table I. The algebraic contributions to each matrix element of the four Dirac and one Majorana matrices will be spelled out explicitly in Ref. [12].

One particularly good numerical choice for the parameters at the SUSY GUT scale is given by

$$\begin{aligned} g_{10}v_u &= 120.3, & g_{10}v_d &= 2.46, & g_{10'}v'_d &= 0.078 \text{ GeV} \\ g_{126}w_u &= 0.314, & g_{126}w_d &= -0.037, & g_{126'}w' &= 0.8 \times 10^{16} \text{ GeV} \\ g_{45_X}u_{45_X}/M &= 0.130, & g_{45_Z}u_{45_Z}/M &= 0.165, & g_1u_1/M &= 0.56, & g_{1'}u_{1'}/M &= -0.026 \\ \phi_{45_X} &= 35^\circ, & \phi_{45_Z} &= \phi_1 = \phi_{1'} = -5^\circ \end{aligned} \tag{10}$$

which reduces the number of independent parameters to 12 and leads to the following mass matrices at the SUSY GUT scale



$$M^U = \begin{pmatrix} -0.0010 - 0.0001i & 0.0053 + 0.0034i & -0.0013 \\ 0.0053 + 0.0034i & 0.314 & 0 \\ -0.0013 & 0 & 120.3 \end{pmatrix} \quad (11a)$$

$$M^D = \begin{pmatrix} -0.0001 & -0.0104 + 0.0004i & -0.0029 - 0.0045i \\ -0.0077 + 0.0018i & -0.036 & 0.078 \\ -0.0033 - 0.0048i & 0.078 & 2.460 \end{pmatrix} \quad (11b)$$

$$M^N = \begin{pmatrix} 0.0030 + 0.0003i & -0.079 - 0.051i & 0.0038 \\ 0.048 + 0.031i & -0.942 & 0 \\ 0.0038 & 0 & 120.3 \end{pmatrix} \quad (11c)$$

$$M^E = \begin{pmatrix} 0.0004 & -0.0020 - 0.0010i & -0.0023 - 0.0045i \\ 0.0060 + 0.0031i & 0.108 & 0.078 \\ -0.0009 - 0.0037i & 0.078 & 2.460 \end{pmatrix} \quad (11d)$$

$$M^R = \begin{pmatrix} (-.075 + .637i) \times 10^9 & (-.139 - .117i) \times 10^{11} & (.106 + .019i) \times 10^{13} \\ (-.139 - .117i) \times 10^{11} & (.454 + .541i) \times 10^{12} & (-.387 - .152i) \times 10^{14} \\ (.106 + .019i) \times 10^{13} & (-.387 - .152i) \times 10^{14} & (.247 - .044i) \times 10^{16} \end{pmatrix} \quad (11e)$$

in units of GeV. The Dirac mass matrix elements appear in the form  $\psi_L^T C^{-1} M (\psi^c)_L$ , while the Majorana matrix elements refer to  $(\psi^c)_L^T C^{-1} M (\psi^c)_L$  with  $\psi_L$  and  $(\psi^c)_L$  each a member of one of the three families of **16**'s. Identical contributions also arise from the transposed Dirac matrices and the right-handed Majorana matrix. As such, the true Yukawa couplings  $G_Y$  are just half the values of the  $g_Y$ 's appearing in (9) and (10).

The masses at the GUT scale can then be found by calculating the eigenvalues of the Hermitian product  $MM^\dagger$  in each case, while the mixing matrices  $V_{CKM}$  and  $V_{lepton}$  can be calculated with the projection operator technique of Jarlskog [13]. After evolving these quantities to the low scale, we find in the quark sector

$$\begin{aligned} m_u(1\text{GeV}) &= 5.0 \text{ (5.1) MeV}, & m_d(1\text{GeV}) &= 7.9 \text{ (8.9) MeV} \\ m_c(m_c) &= 1.27 \text{ (1.27) GeV}, & m_s(1\text{GeV}) &= 169 \text{ (175) MeV} \\ m_t(m_t) &= 150 \text{ (165) GeV}, & m_b(m_b) &= 4.09 \text{ (4.25) GeV} \end{aligned} \quad (12a)$$

where we have indicated the preferred values in parentheses. The mixing matrix is given by

$$V_{CKM} = \begin{pmatrix} 0.972 & 0.235 & 0.0037e^{-i124^\circ} \\ -0.235 & 0.971 & 0.041 \\ 0.012 & -0.039 & 0.999 \\ -0.003i & -0.001i & \end{pmatrix} \quad (12b)$$

Note that  $|V_{ub}/V_{cb}| = 0.090$  with the CP-violating phase  $\delta = 124^\circ$ , while  $m_d/m_u = 1.59$ ,  $m_s/m_d = 21.3$ , cf. Ref. [14]. In the lepton sector we obtain

$$\begin{aligned} m_{\nu_e} &= 0.10 (?) \times 10^{-4} \text{ eV}, & m_e &= 0.44 (0.511) \text{ MeV} \\ m_{\nu_\mu} &= 0.29 (0.25) \times 10^{-2} \text{ eV}, & m_\mu &= 99 (105.5) \text{ MeV} \\ m_{\nu_\tau} &= 0.11 (0.10) \text{ eV}, & m_\tau &= 1.777 (1.777) \text{ GeV} \end{aligned} \quad (13a)$$

and

$$V_{lept} = \begin{pmatrix} 0.998 & 0.050 & 0.038e^{-i122^\circ} \\ -0.036 & 0.873 & 0.486 \\ 0.043 & -0.485 & 0.873 \\ -0.037i & -0.002i & \end{pmatrix} \quad (13b)$$

The heavy Majorana neutrino masses are

$$M_1^R = 0.63 \times 10^9 \text{ GeV}, \quad M_2^R = 0.39 \times 10^{11} \text{ GeV}, \quad M_3^R = 0.25 \times 10^{16} \text{ GeV} \quad (13c)$$

The neutrino masses and mixings are in the correct ranges to explain the nonadiabatic solar neutrino depletion [3] with small mixing and the atmospheric neutrino depletion [4] with large mixing:

$$\begin{aligned} \delta m_{12}^2 &= 8.5 \times 10^{-6} \text{ eV}^2, & \sin^2 2\theta_{12} &= 1.00 \times 10^{-2} \\ \delta m_{23}^2 &= 1.2 \times 10^{-2} \text{ eV}^2, & \sin^2 2\theta_{23} &= 0.72 \end{aligned} \quad (14)$$

For our analysis, the SUSY GUT scale at which the gauge and Yukawa couplings unify was chosen to be  $\Lambda = 1.2 \times 10^{16} \text{ GeV}$ . From (4a) and (11) we find that  $g_{10} = 0.69$ . It is interesting to note that if we equate the  $SO(10)$ -breaking and lepton number-breaking VEV,  $w'$ , with  $\Lambda$ , we find  $g_{126'} = 0.67 \simeq g_{10}$ . Taking into account the remark following (11e), we note the true Yukawa couplings are  $G_{10} \simeq G_{126'} \simeq 0.33$ . If we further equate  $g_1 = g_{10} \simeq g_{126'}$ ,

and  $u_1 = \Lambda$  for the  $U(1)_F$ -breaking VEV, we find  $M = 1.5 \times 10^{16}$  GeV for the masses of the superheavy fermions which condense with their mirrors. These values are all very reasonable.

The  $45_X$  and  $45_Z$  VEVs appear at nearly the same scale,  $2.8 \times 10^{15}$  and  $3.5 \times 10^{15}$  respectively, if one assumes the same Yukawa coupling as above. On the other hand, if these VEVs appear at the unification scale  $\Lambda$  the corresponding Yukawa couplings are smaller than those found above. In either case, a consequence of their non-orthogonal breakings is that  $SU(5)$  is broken down to  $SU(3)_c \times SU(2)_L \times U(1)_Y$  at the scale in question. No further breaking is required until the electroweak scale and the SUSY-breaking scale are reached.

In summary, we have constructed an  $SO(10) \times U(1)_F$  model of the Yukawa interactions with the following features:

(i) The global  $U(1)_F$  family symmetry group singles out a rather simple set of tree diagrams which determines the texture of the generic Dirac and Majorana mass matrices, while the  $SO(10)$  group relates corresponding matrix elements of the up, down, neutrino and charged lepton Dirac matrices to each other.

(ii) The dominant second and third family Yukawa interactions are renormalizable and arise through couplings with Higgs in the  $10$ ,  $10'$  and  $\overline{126}$  representations of  $SO(10)$ . The remaining Yukawa interactions are of higher order and require couplings of Higgs in the  $\overline{126}'$ ,  $1$ ,  $1'$ ,  $45_X$  and  $45_Z$  representations which acquire VEVs near the SUSY GUT scale.

(iii) The Higgs which acquire high scale VEVs break the  $SO(10) \times U(1)_F$  symmetry down to the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  standard model symmetry.

(iv) Although this non-minimal supersymmetric model involves several Higgs representations, the runnings of the Yukawa couplings from the GUT scale to the low-energy SUSY-breaking scale are controlled mainly by the contributions from the  $10$ , as in the minimal supersymmetric standard model.

(v) In terms of 12 input parameters, 15 masses (including the heavy Majorana masses) and 8 mixing parameters emerge. The Yukawa couplings and the Higgs VEVs are numerically feasible and successfully correlate all the quark and lepton masses and mixings in

the scenario which incorporates the nonadiabatic solar neutrino and atmospheric neutrino depletion effects.

We shall elaborate further on the numerical details in a paper now in preparation [12]. Work is also underway to construct a superpotential for the model presented here.

The authors gratefully acknowledge the Summer Visitor Program and warm hospitality of the Fermilab Theoretical Physics Department where much of this research was initiated and carried out. We thank Joseph Lykken for his continued interest and comments during the course of this work. The research was supported in part by the U. S. Department of Energy.

## References

- [1] C. H. Albright and S. Nandi, Phys. Rev. Lett. **73** (1994) 930; Report No. Fermilab-PUB-94/061-T and OSU Report No. 286 (to appear in Phys. Rev. D); and Report No. Fermilab-PUB-94/119-T and OSU Report No. 289 (to be published).
- [2] H. Georgi and C. Jarlskog, Phys. Lett. **86B** (1979) 297; J. A. Harvey, P. Ramond and D. B. Reiss, Phys. Lett. **92B** (1980) 309; H. Arason, D. Castan o, B. Keszthelyi, S. Mikaelian, E. Piard, P. Ramond and B. Wright, Phys. Rev. Lett. **67** (1991) 2933; Phys. Rev. D **46** (1992) 3945; S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. **68** (1992) 1984; Phys. Rev. D **45** (1992) 4192; **46**, (1992) R4793; **47** (1993) R3702; G. F. Giudice, Mod. Phys. Lett. A **7** (1992) 2429; H. Arason, D. Castan o, P. Ramond and E. Piard, Phys. Rev. D **47** (1993) 232; P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. **B406** (1993) 19; A. Kusenko and R. Shrock, Phys. Rev. D **49**, 4962 (1994); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **74** (1995) 2418.
- [3] R. Davis et al., Phys. Rev. Lett. **20** (1968) 1205; in *Neutrino '88*, ed. J. Schnepf et al. (World Scientific, 1988); K. Hirata et al., Phys. Rev. Lett. **65** (1990) 1297, 1301; P. Anselmann et al., Phys. Lett. B **327** (1994) 377, 390; Dzh. N. Abdurashitov et al., Phys. Lett. B **328** (1994) 234.
- [4] K. S. Hirata et al., Phys. Lett. B **280** (1992) 146; and **283** (1992) 446; R. Becker-Szendy et al., Phys. Rev. Lett. **69** (1992) and Phys. Rev. D **46** (1992) 3720; W. W. M. Allison et al., Report No. ANL-HEP-CP-93-32; Y. Fukuda et al., Phys. Lett. B **335** (1994) 237.
- [5] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. D **50** (1994) 1173.
- [6] S. P. Mikheyev and A. Yu Smirnov, Yad Fiz. **42** (1985) 1441 [Sov. J. Nucl. Phys. **42** (1986) 913]; Zh. Eksp. Teor. Fiz. **91** (1986) 7 [Sov. Phys. JETP **64** (1986) 4]; Nuovo Cimento **9C** (1986) 17; L. Wolfenstein, Phys. Rev. D **17** (1978) 2369; **20** (1979) 2634.

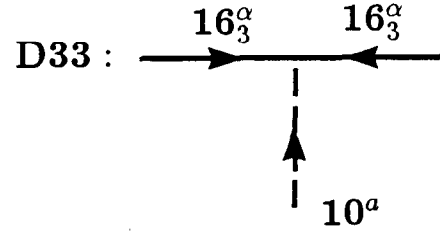
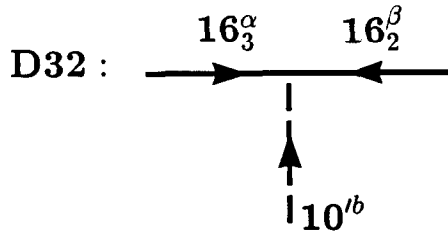
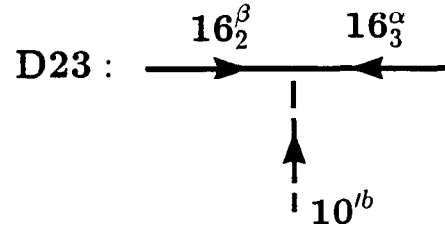
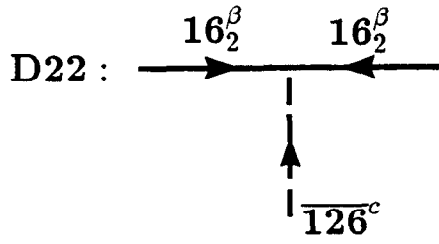
- [7] A. Kusenko, Phys. Lett. B **284** (1992) 390.
- [8] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supersymmetry*, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Prog. Theor. Phys. B **315** (1978) 66.
- [9] The form of  $M^R$  presented here differs somewhat from that in Ref. [1], for more recent data on the atmospheric depletion effect was taken into account.
- [10] For recent use of  $U(1)_F$  symmetry to generate patterns of fermion mass matrices, see L. Ibanez and G. G. Ross, Phys. Lett. B **332** (1994) 100; P. Binetrui and P. Ramond, LPTHE-ORSAY-94-115 preprint; H. Dreiner, G. K. Leontaris, S. Lola and G. G. Ross, Nucl. Phys. B **436** (1995) 461.
- [11] S. Dimopoulos, Phys. Lett. B **129** (1983) 417.
- [12] C. H. Albright and S. Nandi, (in preparation).
- [13] C. Jarlskog, Phys. Rev. D **35** (1987) 1685; **36** (1987) 2138; C. Jarlskog and A. Kleppe, Nucl. Phys. B **286** (1987) 245.
- [14] J. Gasser and H. Leutwyler, Phys. Rep. C **87** (1982) 77.

SU(5) Assignments	VEV Directions			Flipped SU(5) Assignments
	$\mathbf{45}_X$	$\mathbf{45}_Y$	$\mathbf{45}_Z$	
$u, d$	1	1	1	$d, u$
$u^c$	1	- 4	2	$d^c$
$d^c$	- 3	2	- 4	$u^c$
$\nu, \ell$	- 3	- 3	- 3	$\ell, \nu$
$\nu^c$	5	0	6	$e^c$
$e^c$	1	6	0	$\nu^c$

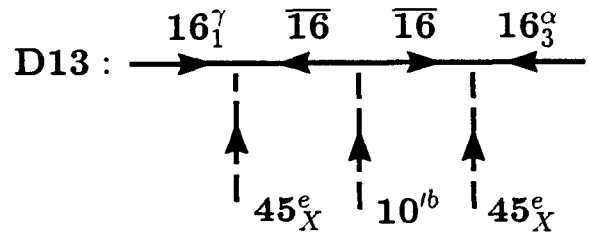
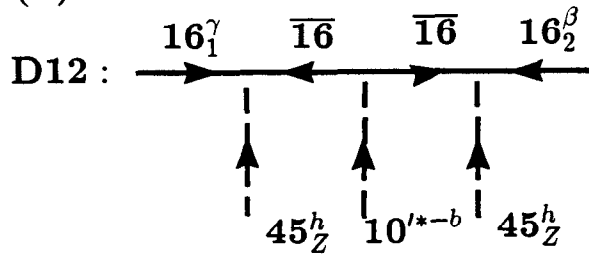
Table I. Couplings of the  $\mathbf{45}$  VEVs to states in the  $\mathbf{16}$ .

Fig. 1. Tree-level diagrams for the (a) renormalizable and (b) leading-order nonrenormalizable contributions to the generic Dirac mass matrix and for the (c) 33 element of the Majorana mass matrix.

(a)



(b)



(c)

